

Add an Imitator, so that capture square for every missing man is unambiguous a) Diagram b) +wRh1 -bRh8

## 1. PRELIMINARY REMARKS:

Whatever the paths and the number of moves, any displacement has a horizontal component [+ right, - left] and a vertical one [+ up, - down]. If Ke1 $\rightarrow$ c5, h=-2 and v=+4. If Qd8 $\rightarrow$ g2, h=+3 and v=-6. These values also express the displacements, always equipollent, of the Imitator. One may add respectively different h and different v. Here -2+3=+1 and +4-6=-2 are respectively the h and v-displacements of the Imitator resulting from the combination "Ke1 $\rightarrow$ c5, Qd8 $\rightarrow$ g2".

1) If H and V are the additions respectively of all the h and all the v of all the pieces moved in a game [for that, one must know the initial and final squares of each piece], H and V are also the horizontal and vertical displacements of the Imitator between its position at the start of the game and its final position. When a diagram shows the final position of the Imitator, one can deduce its starting position by applying the inverse, -H and -V.

2) When calculating H and V, the initial squares may be inverted. Compare "Ke1 $\rightarrow$ c5, Qd8 $\rightarrow$ g2" and "Qd8 $\rightarrow$ c5, Ke1 $\rightarrow$ g2". In the first case, horizontally -2+3=+1, vertically +4-6=-2. In the second, -1+2=+1 and -3+1=-2. Same results! The final squares also may be reversed. Compare "Sb1 $\rightarrow$ f3, Qd1 $\rightarrow$ c7" and "Sb1 $\rightarrow$ c7, Qd1 $\rightarrow$ f3". First, horizontally +4-1=+3, vertically +2+6=+8. Then, +1+2=+3 and +6+2=+8. Same results! But mind some impossible inversions: for example a Bf1 may move to f3 but not to c7.

3) At the start of the game, the Imitator must be on the board, but cannot be on 1, 2, 7 or 8, ranks occupied by the pieces, not on 6 as White has not even a move, and not either on 5 if the first move has to be done by a S! Thus we have  $-7 \le H \le +7$  and  $-4 \le V \le +5$  otherwise the Imitator cannot be on the board at once at the start of the game and in the given position.

## 2 - FORCED DISPLACEMENTS IN a) AND b).

Clearly, there is only one possible capture square for each missing B, as 3 Bs died on c1, c8 and f1 without any displacement!

We arbitrarily consider Sa1 was on b1 and Sa8 on b8 at the start of the game [see 1-2)]. Because these Ss came on a1/a8 before a2xb3/a7xb6, the Rs could never escape from 1st/8th ranks. Thus, the pieces captured by Ps were bQ or bS on b3 and wQ or wS on b6. We have the following forced displacements. Horizontally Ka2=-4, Sa1=-1, Pb3=+1, Kb5=-3, Sa8=-1, Pb6=+1, wSb6+bQb3 or wQb6+bSb3 =-7. Total h(forced)=-14. Vertically Ka2=+1, Sa1=0, Pb3=+1, Kb5=-3, Sa8=0, Pb6=-1, wSb6+bQb3 or wQb6+bSb3 =0. Total v(forced)=-2.

wQb6+bQb3 is illegal because a S must disappear to allow a Q to leave the 1st or 8th rank. wSb6+bSb3 is intentionally omitted, but afterwards anyone may verify that this combination, when legal, does not add anything to all what follows. Here h(forced) is the displacement the less negative. The result h(forced)=-14 implies a positive displacement of the others pieces, so that the Imitator could be on the board at the start of the game [see 1-3)].

3 - IN POSITION a), THE IMITATOR CANNOT BE ON 1st OR 8th RANK!

"Imitator on 1 or 8" is a special case, as Black must immediately uncapture to avoid White retro-stalemate! As retracting a7xb6 is illegal (the bK cannot move back to e8), there are only two possibilities.

An uncapture on b5 would be illegal.

If wQb6+bSb3, a wS would be uncaptured on b5 and h(forced) would become -14-5=-19. The most positive h-displacements of the other pieces would be: bRa=+2 (captured on c8 before bQ and bK left the 8th rank, this being possible only after Sb6-a8 and a7xb6), bQ=+4 (on h), wRh=0 (on h1), wRa=+5 (captured on f1 with wK and wQ on h1 and g1, wRh being already captured). Total H=-19+2+4+0+5=-8. Same result with any other combination of captures on b3,b6,b5 [see 1-2)]. H=-8 is illegal [see 1-3)].

But why not an uncapture on h8?

3 consequences.

1) A wS would be restored because the wQ could never reach h8, that means wQ captured on b6,

2) The Imitator would be on 8, not on 1 since wSh8 could not retro-move,

3) With wSh8, v(forced) would become -2+7=+5. Imitator on 8 implies V=+5 or +4, otherwise the Imitator cannot be on 3 or 4 at the start of the game, as it must be when the first move is a S move [see 1-3)]. This implies for the bQ a 0 or -1 v-displacement, in other words bQ captured on 8 or 7.

Under these conditions, the most positive h-displacements of the pieces not posted in x(forced) would be: wSh8=+1, wRh=0 (captured on h1), wRa=+5 (on f1), bRa=+4 (on e8 after disappearance of bQ and 0-0-0!!!), bQ=0 (on d8, not on e8 since 0-0-0 would be impossible: first the bQ has to leave the 8th rank via a7, which implies bS already on a8). Here H=-14+1+0+5+4+0=-4 looks legal. Regarding the stipulation of the problem, this is very interesting, because there would be a precise capture square for each missing piece. Is there, on 8, a position of the Imitator respecting that?

1) With Imitator c8, these h-displacements of the pieces would not be exactly needed. For sure, the pieces could disappear as mentioned and, with H=-4 and V=+5, -H and -V would show that the Imitator would have been on g3 at the beginning [see 1-1)]. But one of the pieces could be captured as well one square more on the left (for example, bQ on c8 instead of d8), leading to H=-5 and Imitator on h3 at the beginning, which is also legal. With Imitator c8 (or b8), all is not "unambiguous" as required.

2) With Imitator e8 (or g8), these h-displacements would be illegal, since the Imitator would have started form out of the board.

3) IMITATOR d8 would force the captures exactly as follows: wQ on b6, wRa on f1, wRh on h1, wS on h8, bQ on d8, bR on e8, bS on b3! H=-4 and V=+5, Imitator on h3 at the beginning!

But this is simply a try! WITH IMITATOR d8, THE POSITION IS ILLEGAL  $\rightarrow$  after the uncapture Rg8xSh8(Ib8-c8), White is retro-stalemated: Sg6-h8(Ib6-c8) is illegal, since b6 is occupied!

## 4 - SOLUTION a).

Now that the special case "Imitator on 1 or 8" is settled, let us have a more general approach in our search for the most positive h-displacement in compensation for h(forced)=-14.

For the pieces not posted in h(forced), it is: wRh=0, wRa=+5, bRa=+2 (captured on c8 before bQ and bK left the 8th rank), wQ+bS or wS+bQ =+5 (wQ+bS or wS+bQ captured on h). This leads to H=-2. By placing the Imitator on f (Imitator on h at the start of the game), this most positive h-displacement will be inevitable.

1) But the first uncapture (inevitably on h!) cannot be done by a K: with the Imitator on f, no K can reach h. For that, the Imitator must be much more on the left!

2) An uncapture by the bR is compatible with Imitator on f! That means a wS must be restored first on h8, as in the try!

Vertically, we have: v(forced)=-2, wSh8=+7, wRh=wRa=bRa=0, bQ=v (v is unknown for the time being). This leads to V=(v+5). The question is now: is it possible to have the Imitator on f6, f5, f4 or f3, so that v is known without ambiguity?

1) With Imitator f6, V=+3 or +2 is needed, otherwise the Imitator cannot be on 3 or 4 at the beginning, as it must be. That means v=-2 or -3, in other words: bQ captured on h6 or h5. 2) With Imitator f5, V=+2 or +1, v=-3 or -4, bQ captured on h5 or h4.

3) With Imitator f4, V=+1 or 0, v=-4 or -5, bQ captured on h4 or h3.

Regarding the stipulation, these duals are not satisfactory.

4) With Imitator f3, V= 0 or -1, v=-5 but not -6, since the bQ could not disappear on h2, as this square was always occupied!

In this case, the steps of the retro-genesis are: 1) the bR uncaptures the wS on h8 2) the wS uncaptures the bQ on h3 3) bK and bQ return to 8 4) the bP uncaptures the wQ on b6 5) wK and wQ return to 1 6) the wP uncaptures the bS on b3 7) the Ss uncapture the Rs and Bs and return home!

Thus, solution a) is IMITATOR ON f3, forcing the captures as follows: wQ on b6, wRa on f1, wRh on h1, wS on h8, bQ on h3, bR on c8, bS on b3! H=-2 and V= 0, Imitator on h3 at the start of the game!

5 - IN POSITION b), THE IMITATOR CANNOT BE ON 1st OR 8th RANK!

"Imitator on 1 or 8" remains special. The wK cannot retro-move immediately, the bK can, but only on 5th rank. An uncapture is possible on h1 instead of h8. No retro-stalemate this time, and therefore the first uncapture is not necessarily immediate. Theoretically, the bK himself can uncapture on different squares of 5th rank.

An uncapture on 5th rank would be illegal.

To v(forced)=-2, we add +4 for the v-displacement of a wQ or wS restored on 5th rank. Moreover, the bS or bQ not captured on b3 would have a 0 or negative v-displacement. Thus, it would be impossible to have V=+5 or +4 as needed if the Imitator is presently on 8. Only a Sb5 retro-moving on a7 could allow the Imitator to leave the 1st rank. But an uncapture directly on b5 would lead to the h-illegality already noticed in position a). Simply, instead of H=-8 we would have H=-9 because the wRh is now present and the wRa would disappear on e1 instead of f1.

Theoretically, the wQ (uncaptured on 5) could uncapture the bS on b5. But this would lead to V=-2+4-3=-1, which is an illegal result since, with Imitator on 1st rank, we must have V=-2 or -3!

But why not an uncapture on 1st rank?

To v(forced)=-2, we add -7 for the bQ or bS uncaptured on 1st rank. Even by adding +7 (captured on 8) for the wS or wQ not captured on b6, we cannot have H=+5 or +4 as needed if the Imitator is presently on 8.

With Imitator on 1, V=-2 or -3 is needed, so that the previous white piece would have vanished on the 8th or 7th rank. In case of bQ restored on 1, no black piece could uncapture on the 8th or 7th rank because the Imitator could not leave the 1st rank! Only a bS could be restored on 1!

Under these conditions, the most positive h-displacements of the pieces not posted in h(forced) would be: bSh1=+1, bRh=0 (captured on h8), bRa=+2 (on c8 before bQ and bK left the 8th rank), wRa=+4 (on e1, wRh wK and wQ being at this moment on h1 g1 and f1), wQ=+1 (on e8). Here H=-14+1+2+4+1=-6 looks valid.

Thus, IMITATOR b1 would force the captures as follows: wQ on e8, wR on e1, wS on b6, bQ on b3, bRa on c8, bRh on h8, bS on h1! H=-6 and V=-2, Imitator starting from h3! This time, no retro-stalemate as in the try of position a). With Ib1, one can successively retromove for example: Kc5 Rg1(bSh1) Sg3 Ka3 Kd5 (Ib4) ... etc ... The steps of the retro-genesis would be: 1) the wR uncaptures the bS on h1 2) the bS uncaptures the wQ on e8 3) wK and wQ return to 1 4) the wP uncaptures the bQ on b3 5) bK and bQ return to 8 6) the bP uncaptures the wQ on b6 7) the Ss uncapture the Rs and Bs and return home! But this is a try once more! WITH IMITATOR b1, THE POSITION IS ILLEGAL  $\rightarrow$  the steps 1) 2) 3) are actually manageable, but the return to 1st rank of wK+wQ is only possible with bK+bS on 8th rank, the Imitator being inevitably carried on 3rd rank. The 4th step cannot be achieved, the retro-move a2xb3 being illegal with Imitator on 3rd rank!

## 6 - SOLUTION b).

This time the Imitator, for example on f as in Solution a), can be carried toward the left by retro-moving the wR. Then a K may reach h and uncapture on h! Let us have again a more general approach in our search for the most positive h-displacement in compensation for h(forced)=-14.

For the pieces not posted in h(forced), it is: wRa=+4 (captured on e1), bRa=+2 (on c8), bRh=0 (on h8), wQ+bS or wS+bQ =+5 (wQ+bS or wS+bQ captured on h). This leads to H=-3. Vertically: wRa=bRa=bRh=0, wQ+bS or wS+bQ =v (this time, v is the v-displacement of the couple of pieces captured on h!). By addition with v(forced)=-2, we have V=(v-2). H=-3 means: by placing the Imitator on e (Imitator on h at the start of the game), these most positive h-displacements will be inevitable. Is it possible to have the Imitator on e6, e5, e4 or e3, so that v is known without ambiguity? In other words, is it possible to have two precises

pieces precisely splitted on two precise squares?

1) With Imitator e3, V= 0 or -1 is needed, otherwise the Imitator cannot start from 3rd or 4th rank as it must! Thus, v=+2 or +1. This is possible in different manners: wSh4+bQh6, wQh5+bSh5, etc...

2) With Imitator e4, v=+3 or +2: wQh5+bSh6, bSh5+wQh6, etc...

3) With Imitator e5, v=+4 or +3: wQh4+bSh8, wSh6+bQh6, etc...

Regarding the stipulation, these duals are not satisfactory.

4) With Imitator e6, v=+5 or +4. Only 4 combinations, as no Q could reach h8.

4.1) wSh8+bQh6 is illegal. The first piece to be uncaptured is the bQ. With le6, whatever the retro-moves of wR+bK, the wK cannot reach h6!

4.2) wSh8+bQh5 is illegal. The wK cannot reach h5!

4.3) wQh6+bSh8 is illegal. The bK must reach h6 to uncapture the wQ. Whatever the paths, this leads to White retro-stalemate! Example: Kc5(If6) Rf1(Id6) Kd5(Ie6) Re1(Id6) Ke5(Ie6) Rd1(Id6) Kf5(Ie6) Rc1(Id6) Kg5(Ie6) Kb1(If5) Kh6(Ig6) retro-stalemate!

4.4) wQh5+bSh8 ... No problem, the bK can reach h5!

The steps of the retro-genesis are: 1) the bK uncaptures the wQ on h5 2) wK and wQ return to 1 3) the wP uncaptures the bS on b3, wSa1 is freed 4) wSa1 uncaptures the bS on h8 5) bK and bQ return to 8 6) the bP uncaptures the wS on b6 7) the Ss uncapture the Rs and Bs and return home!

Thus, solution b) is IMITATOR ON e6, forcing the captures as follows: wQ on h5, wRa on e1, wS on b6, bQ on b3, bRa on c8, bRh on h8, bS on h8! H=-3 and V=-2, Imitator on h4 at the start of the game!

7 - FINALLY ...

This is a problem with a very original stipulation and 2 solutions at once similar and different. The pieces captured in a) on b are captured in b) on h, and vice versa! Also good unity because, in both positions, there is a very tempting try!

Here are 2 retro-games showing the legality of what has been calculated in the solutions. The position of the Imitator is given every 5 retro-moves.

POSITION a) => (If3) 62.Rh8-g8(Sh8) Ka2-a3 61.Kb5-a6 Sh8-g6 60.Ka6-a7 Ka3-a4 59.Ka7-b8 Sg6-f4 58.Kb8-c8 Sf4-h3 (If3) 57.Kc8-d8 Sh3-f4(Qh3) 56.Qh3-h5 Ka4-a3

55.Qh5-f5 Ka3-a2 54.Qf5-e5 Ka2-b1 53.Qe5-f5 Kb1-c1 (le3) 52.Qf5-d5 Kc1-d1 51.Qd5-c5 Kd1-e1 50.Qc5-b5 Ke1-f1 49.Qb5-a6 Kf1-g1 48.Qa6-a7 Kg1-h1 (le5) 47.Qa7-b8 Sf4-d3 46.b6-a7(Qb6) Qb6-a5 45.Qb8-c8 Sd3-f4 44.Qc8-b8 Qa5-a2 43.Qb8-c8 Sf4-d5 (lc4) 42.Qc8-b8 Qa2-b1 41.Qb8-c8 Sd5-b6 40.Qc8-b8 Qb1-g1 39.Qb8-c8 b3-a2(Sb3) 38.Sb3-c5 Sb6-a4 (lf3) 37.Sc5-a6 Sa1-b3 36.Sa8-b6 Sb3-a5 35.Qc8-b8 Sa5-b3 34.Qb8-a8 Sb3-d4 33.Kd8-c8 Sd4-b3 (lc4) 32.Kc8-b8 Sb3-c5 31.Sb6-d5 Sc5-b3 30.Sd5-e3 Sb3-a5 29.Se3-f1 Sa4-b6 28.Sf1-e3(Rf1) Sb6-d5 (lg4) 27.Kb8-c8 Rf1-a1 26.Kc8-d8 Qg1-d1 25.Kd8-e8 Kh1-g1 24.Qa8-d8 Sa5-c6 23.Se3-g4 Kg1-f1 (lg6) 22.Rg8-h8 Kf1-e1 21.Sg4-e3 Ra1-b1 20.Se3-f1 Rb1-a1 19.Sf1-g3(Bf1) Sd5-b6 18.Sg3-h1 Sb6-c8 (lg6) 17.Rh8-g8 Sc8-b6(Rc8) 16.Sh1-g3(Rh1) Sc6-d4 15.Rg8-h8 Sd4-f3 14.Sg3-f5 Ra1-b1 13.Sf5-d4 Rb1-a1 (le4) 12.Sd4-b3 Sf3-e5 11.Sb3-c1 Ra1-b1 10.Sc1-b3(Bc1) Se5-f3 9.Sb3-c5 Sf3-g1 8.Rc8-b8 Sb6-c8 (lf5) 7.Sc5-e4 Rb1-a1 6.Se4-f6 Sc8-b6(Bc8) 5.Rb8-a8 Sg1-f3 4.Rh8-g8 Sb6-c4 3.Rg8-h8 Sc4-a3 (ld3) 2.Sa6-b8 Sf3-g1 1.Sf6-g8 Sa3-b1 (lh3)!

POSITION b) => (le6) 66.Kb5-c5 Rh1-e1 65.Kc5-d5 Re1-d1 64.Kd5-e5 Rd1-c1 63.Ke5-f5 Rc1-d1 62.Kf5-g5 Rd1-c1 (le6) 61.Kg5-h5 Rc1-e1 60.Kh5-g5(Qh5+) Qh5-f3 59.Kg5-h6 Qf3-c3 58.Kh6-q5 Re1-h1 57.Kq5-f5 Qc3-b4 (lc5) 56.Kf5-e5 Qb4-a3 55.Ke5-f5 Ka2-b1 54.Kf5-e6 Kb1-c1 53.Ke6-d6 Kc1-d1 52.Kd6-d5 Kd1-e1 (ld3) 51.Kd5-d6 Ke1-f1 50.Kd6-c6 Kf1-g1 49.Kc6-b5 Kg1-f1 48.Kb5-a6 Kf1-e1 47.Ka6-a7 Qa3-a2 (la4) 46.Ka7-b8 Qa2-b1 45.Kb8-c8 b3-a2(Qb3) 44.Qb3-a3 Sa1-b3 43.Kc8-b8 Sb3-d4 42.Kb8-a7 Sd4-f3 (le4) 41.Ka7-a6 Sf3-e5 40.Ka6-a5 Se5-g6 39.Ka5-a4 Sg6-h8 38.Ka4-b4 Sh8-g6(Rh8) 37.Rh8g8 Sg6-h8 (lg6) 36.Kb4-a4 Sh8-g6(Sh8) 35.Ka4-a5 Sg6-e5 34.Ka5-a6 Se5-d3 33.Ka6-a7 Ke1-f1 32.Ka7-b8 Kf1-g1 (le5) 31.Kb8-c8 Qb1-d1 30.Sh8-g6 Qd1-e1 29.Qa3-a6 Sd3-c1 28.Sg6-e5 Qe1-f1 27.Qa6-a7 Sc1-b3 (le6) 26.Se5-d3 Sb3-c5 25.Sd3-e1 Sc5-a4 24.Qa7b8 Sa4-c3 23.b6-a7(Sb6+) Sb6-d5 22.Se1-d3(Re1) Re1-c1 (le5) 21.Kc8-d8 Sd5-b6 20.Kd8-e8 Rc1-a1 19.Qb8-d8 Qf1-d1 18.Sd3-c1 Sb6-c8 17.Rg8-h8 Sc8-b6(Rc8) (lc4) 16.Sc1-d3(Bc1) Sc3-b1 15.Rc8-b8 Sb6-c8 14.Sa8-b6 Sb1-c3 13.Sb6-c4 Kg1-f1 12.Sd3-e5 Sc8-b6(Bc8) (le4) 11.Se5-c6 Kf1-e1 10.Sc4-e3 Rh1-f1 9.Rb8-a8 Rf1-g1 8.Sc6-b8 Sb6-d5 7.Se3-f1 Rg1-h1 (le3) 6.Sf1-g3(Bf1) Sc3-b1 5.Sg3-h5 Sd5-c3 4.Sh5-f6 Sc3-e4 3.Sf6-d5 Se4-g5 2.Sd5-f6 Sg5-f3 1.Sf6-g8 Sf3-g1 (lh4)!